

Wednesday 29 June 2016 – Morning

A2 GCE MATHEMATICS

4731/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

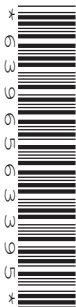
OCR supplied materials:

- Printed Answer Book 4731/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

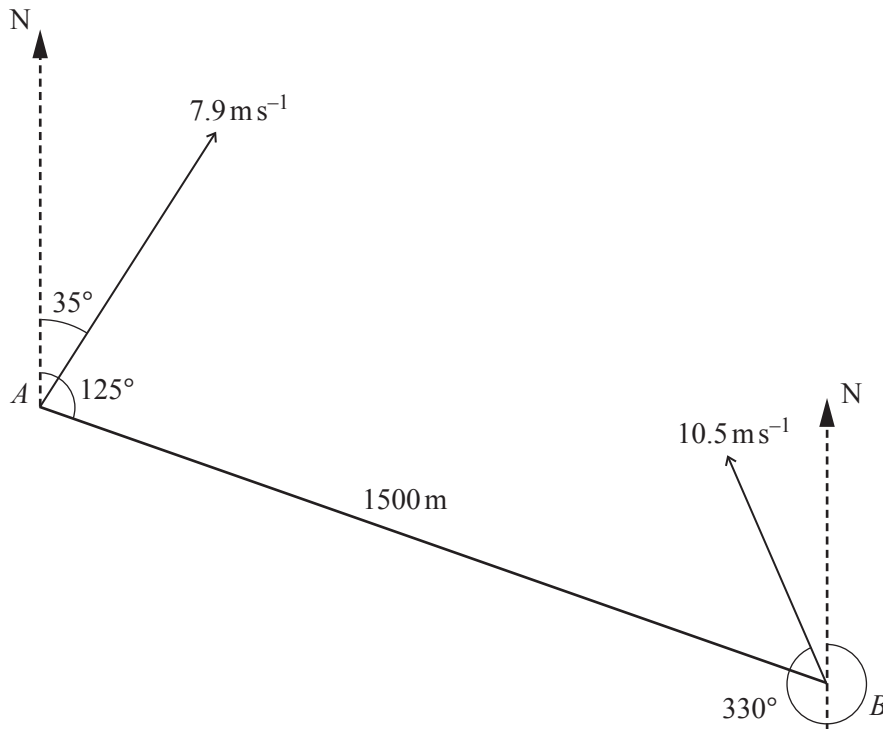
INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer **all** the questions.

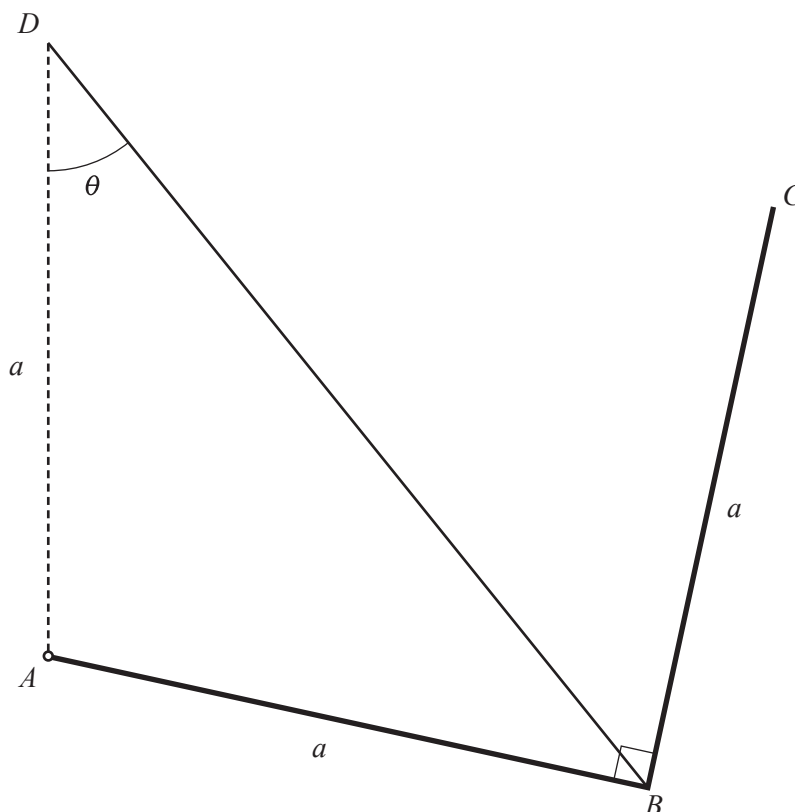
- 1 A uniform square lamina, of mass 5 kg and side 0.2 m, is rotating about a fixed vertical axis that is perpendicular to the lamina and that passes through its centre. A couple of constant moment 0.06 N m is applied to the lamina. The lamina turns through an angle of 155 radians while its angular speed increases from 8 rad s^{-1} to $\omega \text{ rad s}^{-1}$. Find ω . [4]

2



Boat A is travelling with constant speed 7.9 m s^{-1} on a course with bearing 035° . Boat B is travelling with constant speed 10.5 m s^{-1} on a course with bearing 330° . At one instant, the boats are 1500 m apart with B on a bearing of 125° from A (see diagram).

- (i) Find the magnitude and the bearing of the velocity of B relative to A . [5]
- (ii) Find the shortest distance between A and B in the subsequent motion. [2]
- (iii) Find the time taken from the instant when A and B are 1500 m apart to the instant when A and B are at the point of closest approach. [2]



Two uniform rods AB and BC , each of length a and mass m , are rigidly joined together so that AB is perpendicular to BC . The rod AB is freely hinged to a fixed point at A . The rods can rotate in a vertical plane about a smooth fixed horizontal axis through A . One end of a light elastic string of natural length a and modulus of elasticity λmg is attached to B . The other end of the string is attached to a fixed point D vertically above A , where $AD = a$. The string BD makes an angle θ radians with the downward vertical (see diagram).

- (i) Taking D as the reference level for gravitational potential energy, show that the total potential energy V of the system is given by

$$V = \frac{1}{2}mga(\sin 2\theta - 3 \cos 2\theta) + \frac{1}{2}\lambda mga(2 \cos \theta - 1)^2 - 2mga. \quad [5]$$

- (ii) Given that $\theta = \frac{1}{4}\pi$ is a position of equilibrium, find the exact value of λ . [4]

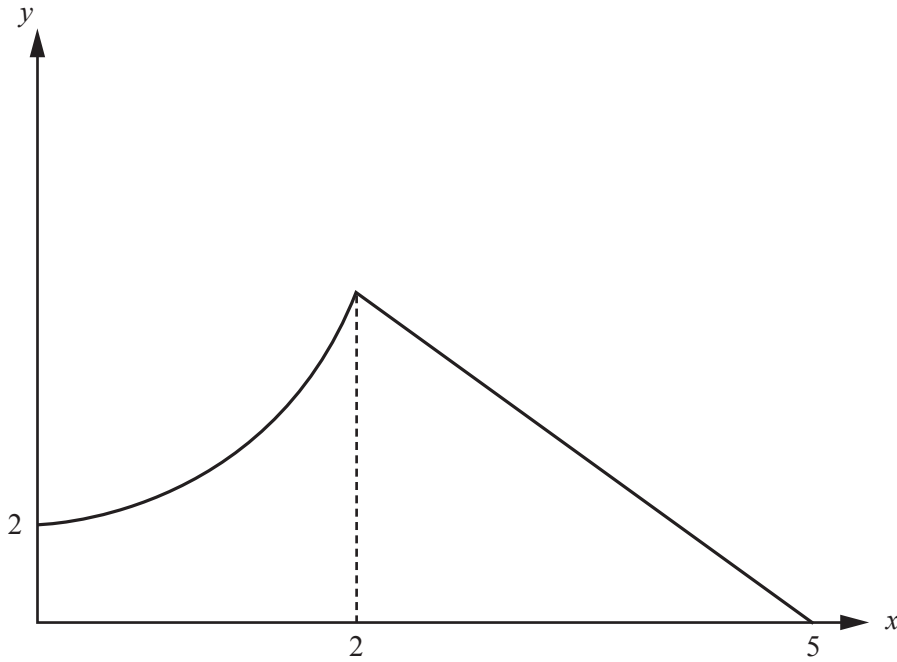
- (iii) Find $\frac{d^2V}{d\theta^2}$ and hence determine whether the position of equilibrium at $\theta = \frac{1}{4}\pi$ is stable or unstable. [4]

- 4 The region bounded by the curve $y = 2e^{\frac{1}{2}x}$ for $0 \leq x \leq 2$, the x -axis, the y -axis and the line $x = 2$, is occupied by a uniform lamina.

(i) Find the exact value of the y -coordinate of the centre of mass of the lamina. [6]

As shown in the diagram below, a uniform lamina occupies the closed region bounded by the x -axis, the y -axis and the curve $y = f(x)$ where

$$f(x) = \begin{cases} 2e^{\frac{1}{2}x} & 0 \leq x \leq 2, \\ \frac{2}{3}(5-x)e & 2 \leq x \leq 5. \end{cases}$$



(ii) Find the exact value of the x -coordinate of the centre of mass of the lamina. [7]

5 A uniform rod AB has mass $2m$ and length $4a$.

- (i) Show by integration that the moment of inertia of the rod about an axis perpendicular to the rod through A is $\frac{32}{3}ma^2$ [4]

The rod is initially at rest with B vertically below A and it is free to rotate in a vertical plane about a smooth fixed horizontal axis through A . A particle of mass m is moving horizontally in the plane in which the rod is free to rotate. The particle has speed v , and strikes the rod at B . In the subsequent motion the particle adheres to the rod and the combined rigid body Q , consisting of the rod and the particle, starts to rotate.

- (ii) Find, in terms of v and a , the initial angular speed of Q . [4]

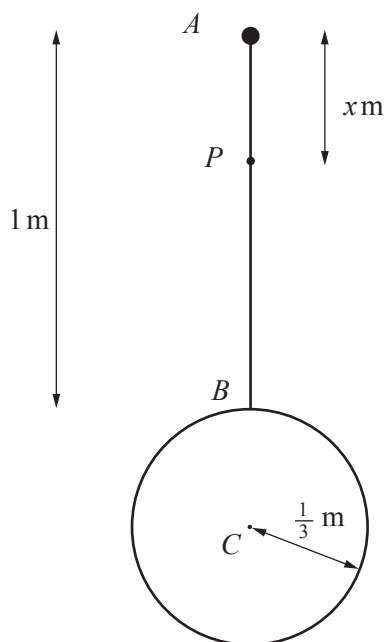
At time t seconds the angle between Q and the downward vertical is θ radians.

- (iii) Show that $\dot{\theta}^2 = k\frac{g}{a}(\cos\theta - 1) + \frac{9v^2}{400a^2}$, stating the value of the constant k . [4]

- (iv) Find, in terms of a and g , the set of values of v^2 for which Q makes complete revolutions. [2]

When Q is horizontal, the force exerted by the axis on Q has vertically upwards component R .

- (v) Find R in terms of m and g . [4]



A compound pendulum consists of a uniform rod AB of length 1 m and mass 3 kg , a particle of mass 1 kg attached to the rod at A and a circular disc of radius $\frac{1}{3}\text{ m}$, mass 6 kg and centre C . The end B of the rod is rigidly attached to a point on the circumference of the disc in such a way that ABC is a straight line. The pendulum is initially at rest with B vertically below A and it is free to rotate in a vertical plane about a smooth fixed horizontal axis passing through the point P on the rod where $AP = x\text{ m}$ and $x < \frac{1}{2}$ (see diagram).

- (i) Show that the moment of inertia of the pendulum about the axis of rotation is $(10x^2 - 19x + 12)\text{ kg m}^2$. [6]

The pendulum is making small oscillations about the equilibrium position, such that at time t seconds the angular displacement that the pendulum makes with the downward vertical is θ radians.

- (ii) Find the angular acceleration of the pendulum, in terms of x , g and θ . [4]
- (iii) Show that the motion is approximately simple harmonic, and show that the approximate period of oscillations, in seconds, is given by $2\pi\sqrt{\frac{20x^2 - 38x + 24}{(19 - 20x)g}}$. [2]
- (iv) Hence find the value of x for which the approximate period of oscillations is least. [3]

END OF QUESTION PAPER

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